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Graphical Markov Models as an alternative to SEM

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- Overview of SEM in Marketing Research
- Introduction to Graphical Markov Models and their relation to SEM (and PLS)
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The Role of SEM in Marketing Research (1)

Structural Equation Modeling (SEM) is a very popular tool for theory testing in marketing and behavioral sciences (Steenkamp & Baumgartner, 2000; Baumgartner & Homburg, 1996; Hulland et al., 1996)

- SEM manage the inclusion of multiple endogenous / exogenous constructs
- SEM account for measurement error in the latent constructs
- Sound theoretical assumptions are confronted with their "fit" with directly observable data (both structural and measurement model)
- Two SEM-philosophies: Covariance-based SEM vs. Variance-based partial least squares SEM (cf. Hair et al., 2012)

The Role of SEM in Marketing Research (2)

- Underlying theoretical justification of SEM models are not always so sound (sometimes they are very weak)
- SEM practice often degenerates to an "exploratory device" for identifying "best" model fitting empirical data (in particular when it comes to "adjust" the measurement models)
- Formal assumptions:

	Cov.-based **SEM**	Var.-based SEM (**PLS**)
Assumptions	multivariate Norm.	no distributional assumptions (appl. for nom., ord. & cont.)
Estimation	ML (or GLS/WLS)	Componentwise
Models	multivariate	uni- & multivariate
Meas. Mod.	factor scores	factor scores
Error Corr.	Bootstrap	Bootstrap
R-packages	lavaan, sem, ...	semPLS, pls, plspm, ...

Graphical Markov models (1)

The approach based on graphical Markov models (in particular DAG) imposes less rigorous assumptions.

- ... are multivariate statistical models, where a graph G ($G = (V, E)$) describes independence statements in the joint distribution
  - **V**: r.v. are denoted by nodes (discrete or continuous)
  - **E**: cond. association parameters in the distribution are represented by edges

A Markovian model is equivalent to a recursive model in SEM.

Example:

$$Z \rightarrow Y \leftarrow X$$

cond. independence rel. $X \perp Z | Y$

$$f(x, y, z) = f(x) \underbrace{f(y|x) f(z|y)}_{linear\ \ regressions}$$

Graphical Markov models (2)

- An initial sequence of the r.v. is defined by (e.g.) research hypotheses (**dependence chain** or joint response chain graph $\mathcal{G} = \{V(1), \ldots, V(p)\}$, i.e., an ordered disjoint partitioning of V)

Example:

$V(1)$		$V(2)$		$V(3)$		$V(4)$
A	$\leftarrow$	B, C	$\leftarrow$	D, E	$\leftarrow$	F, G, H

- All variables are assigned to a higher order component of G are considered conditionally on the prior components
- The density factorizes to

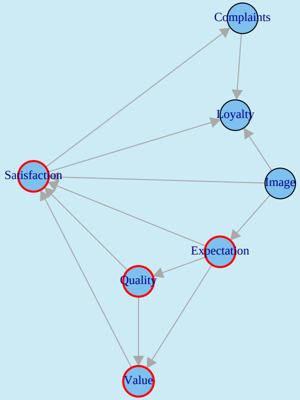
$$f_V = f_{V_p|V_{p-1}\ldots V_1} \cdot f_{V_{p-1}|V_{p-2}\ldots V_1} \cdot \ldots \cdot f_{V_2|V_1} \cdot f_{V_1}$$

Graphical Markov models (3)

- If the variables can be ordered in a recursive response structure we call the **graph a directed acyclic graph** $\mathcal{G}_{dag}$ (variables are neither explanatory nor responses for itself).
- The density f admits a recursive factorization according to $\mathcal{G}$ (y is a random vector)
$$f(y) = \prod_{p \in V} f(y_p | y_{pr(p)})$$
- Models of this type can be constructed via a set of **univariate cond. models**.

The ECSI (European Customer Satisfaction Index) Study (1)

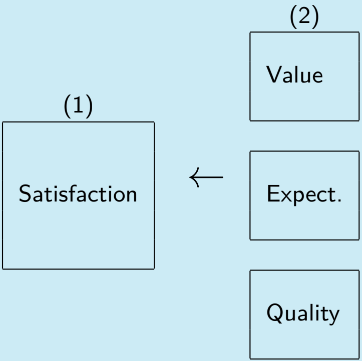
Ex.: Structural model describing causes and consequences of customer satisfaction.



(cf. Tenenhaus et al., 2005)

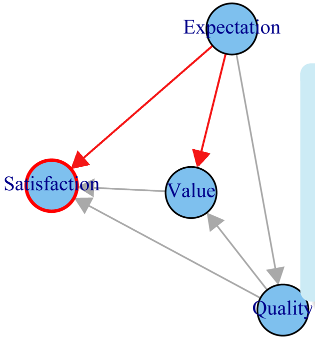
The ECSI (European Customer Satisfaction Index) Study (2)

dependence chain



- First block: all variables in the second block are regressed as independent ones on satisfaction
- Second block: all variables are alternatingly considered as dependent and independent

The European Customer Satisfaction Index (3)



- SEM & PLS: same results (cf. Tenenhaus et al., 2005)
- DAG: same results $f(s,e,v,q) = f(s|v,q)f(v|q)f(e|q)f(q|v,e)$
- Automatic model selection only in DAG (Coefficients for Expectation are very small, $p > 0.05$)

DAG advantages:

- Equation parameters are regression coefficients (interpretation of structure in terms of independencies).
- No overparametrization and consequently no problems of identification.
- General results are available to read directly off the graph of the model.

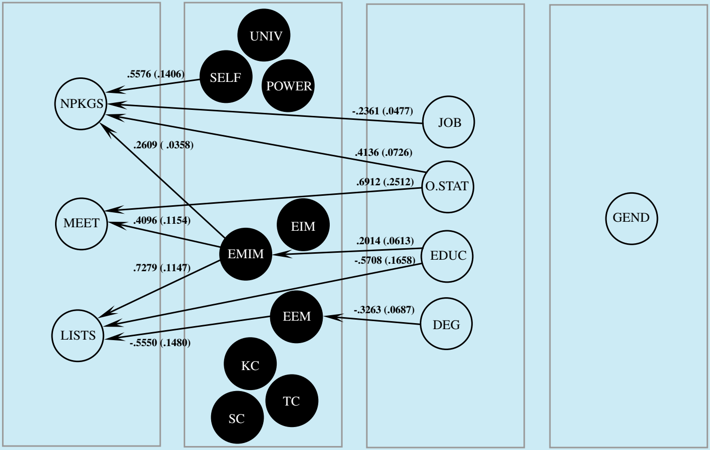
The CRAN Motivation Survey

- Conducted in April/May 2010 to study software developers' motivation to contribute to the OSS project the R-project for statistical computing.
- 4,274 authors of R packages were contacted via Email (CRAN, R-Forge, Bioconductor).
- 782 contributors completed the questionnaire consisted of 120 items (incl. different forms of individual participation and potential motivational drivers).

Dependence chain: Initial ordering of the response (1), intermediate (2 and 3) and purely explanatory (4) variables

(1)		(2)		(3)		(4)
NPKGS MEET (1 = Yes) LISTS (1 = Yes)	$\leftarrow$	UNIV POWER SELF EEM EMIM EIM TC KC SC	$\leftarrow$	JOB (1 = academics) O.STAT (1 = full) EDUC (1 = statistics) DEG (1 = postdoc)	$\leftarrow$	GEND

Model graph of the final graphical chain model. Binary variables are depicted as circles and numerical variables as dots (NPKGS is a discrete poisson distributed variable). Arrows indicate significant relationships.



Summary & Discussion

- The presented kind of Markovian models is particularly interesting for social and behavioral sciences (observational studies, complex multivariate dependencies, existing substantive knowledge)
- Combination of graphical Markov model technique, model building and methods for scaling provide a useful alternative to SEM.
- Only an ordinal structure behind the model has to be specified (no theoretical restrictions on the form of the conditional distributions)
- Variable of mixed measurement scale types can be modeled both within and between levels.

Literature

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Appendix - Model Selection (1)

- By the model selection algorithm of Cox & Wermuth (1996) (heuristic based on backward and forward selection)
- At each step a variable is regressed on all variables belonging to a chain component with a lower number (**univariate conditional regression**)
- Performed for every (potential) response variable to break up complex structures into tractable subcomponents

Non-recursive linear models in SEM are equivalent to block recursive regression models (Lauritzen & Wermuth, 1990)

Appendix - Model Selection (2)

Step 1

Screening for interactions and nonlinear relations by forward selection (full model)

$\Downarrow$

Regression based on main effects (+ nonlinear terms and/or interaction)
using backward selection strategy leads to a reduced model

$\Downarrow$

Step 2

Check for interactions and nonlinear relations based on the reduced model, again backward selection leads to a even more reduced model

$\Downarrow$

Check for interactions and nonlinear terms, again backward selection leads to the finally selected model

Appendix - The CRAN Motivation Survey Measurement Model

- Latent variables are estimated by the use of **item response theory**.
  - Each subject is mapped on a latent dimension by estimating a person parameter (corresponds to factor scores from FA).
  - Different methods are possible (for an overview see de Ayala, 2008)
- additionally: variables are corrected for additive measurement error by the use of **simulation extrapolation method** (SIMEX, Cook & Stefanski, 1994)